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A SIR Transmission Model of Political Figure Fever

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ABSTRACT

In this paper, we present early results of dynamical spread of the voters in political figure fever transmission model. The dynamics of the political figure fever is studied by a SIR compartmental model involving ordinary differential equations for three class of voters; susceptible, infectious, and recovered. The population is assumed constant. From analytical results, we investigated two equilibrium points and their local stability have been discussed based on the basic reproduction number. Numerical simulation is also performed to support the analytical results. From numerical results, the solutions converge to each equilibrium states. It can be seen that when there is the higher boredom rate between voter populations, the infected individual by political figure will decrease faster.

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INTRODUCTION

Mathematical modeling is the process of constructing, testing, and improving mathematical models, which are representations of systems or hypotheses in a mathematical language. Many physical and biological phenomena are presented via mathematical modeling. Mathematical models are used, particularly not only in the natural sciences (such as physics and biology) and engineering disciplines but also in the social sciences (such as political science).

Mathematical models have become important tools in analyzing the spread and control of infectious diseases and plans for defense against disease attacks, especially when combined with powerful, modern computer methods for analyzing and/or simulating the models. Mathematical models can sharpen our understanding of fundamental processes, compare alternative policies and interventions, help make decisions, prepare responses to disease attacks, provide a guide for training exercises and scenario development, guide risk assessment, and predict future trends. There is no best model, only better models.

Mathematical models are useful tools for studying the dynamics of the spread of infectious diseases. The most widely used mathematical models for infectious diseases are called SIR transmission models. Kermack and McKendrick in 1927 proposed the first SIR epidemic model (Kermack, W.O. and A.G. McKendrick, 1927). From this simple model, there have been many researchers that have numerically and analytically analyzed infectious disease model. Furthermore, many studies have developed more complex model and this model is applied to another field, for example in politics. A simple mathematical model for the spread of two political parties was discussed in (Misra, A.K., 2012), an epidemiological approach to the spread of political third parties was discussed in (Peterson, I., 1991), and stability of equilibria in multi-party political system was discussed in (Romero, D.M., 2011). These researches discuss about transmission model in politics related to political parties, not to political figures. In this research, a SIR transmission model of the political figure fever will be constructed. Dynamic of political processes are modeled by deterministic ordinary differential equations (ODE). Deterministic ODE is solved to explain and describe behavior of a system. The first part of this paper constructs a transmission model of political figure fever, the second part analyzes the model, and the last section simulates the model.

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Transmission Model of Political Figure Fever:

Mathematical modeling is a process to build a mathematical model for describing dynamical change of a system. From real data with some assumptions and hypothesis, a mathematical model can be formulated. This model can be analyzed and get a conclusion to predict the results. This prediction will be verified based on real data. For better model, some assumptions can be removed.

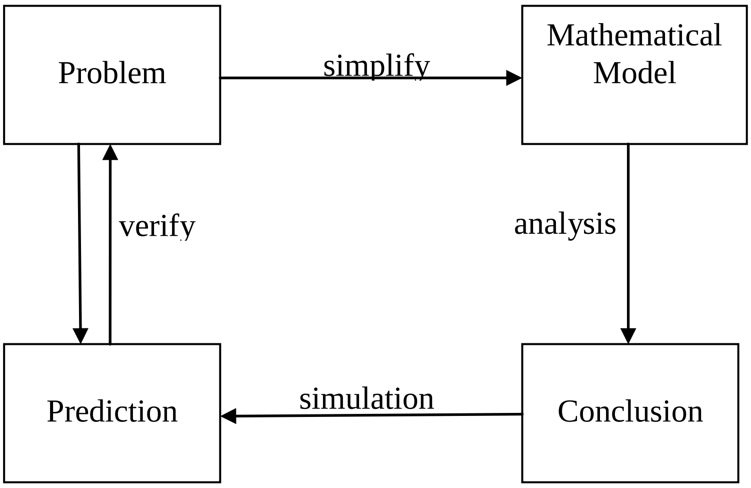


Fig. 1: Modeling process (Brauer, F., 2008).

One common epidemiological model is the SIR model for the spread of disease, which consists of a system of three differential equations that describe the changes in the number of susceptible, infected, and recovered individuals in a given population. The SIR model has played a major role in mathematical epidemiology. In this model, a population is divided into three groups: the susceptibles class S, the infectious (infected) class I, and the removed (recovered) class R. The total population is $N = S + I + R$. Total population is assumed constant.

The susceptibles are those who are not infected, not immune, and they are able to catch the disease. The infected are those who are infected and can transmit the disease to susceptible individuals. The time they spend in the infected compartment is the infectious period, after which they enter the recovered compartment. The recovered are those who have been infected and assumed to be immune for life (recovered or dead). We ignore any subdivisions of the population by age, sex, mobility, or other factors, although such distinctions are obviously of importance.

In epidemiology, the word "epidemic" has a technical meaning: it is a situation in which the number of infective increases from the initial value. The condition for an epidemic follows directly from the equation $\frac{dI}{dt} > 0$. Here, we are much more interested in the time course caused by the introduction of some infectives, and in the final asymptotic state of the system. Some typical questions of interest are:

1. How many susceptibles become infected?
2. What is the peak number of infectives?
3. When does the epidemic peak?
4. How does the time course depend on the initial number of infectives?

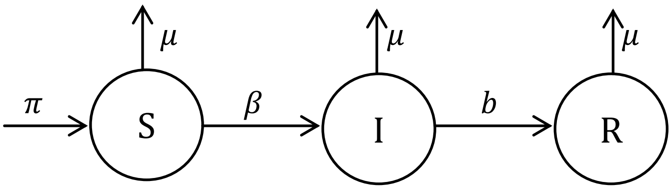


Fig. 2: Compartmental model.

Compartmental model in figure 2 describes the transitions of individuals from S to I to R. All parameters are assumed positive constant with the following interpretation: π is the number of individuals enter into the susceptible voters class, μ is the rate of individuals exit from the system (for example by migration or die) of each class, β is the transmission rate of susceptibles and infectious voters (infectious voters try to convince susceptibles to support their political figure), and b is the boredom rate to political figure of infectious voters class. A compartmental (deterministic) model here offers a functional means of obtaining adequate knowledge of the dynamics of a population when the population is sufficiently large (Brauer, F. and Castillo-Chavez, 2001).

The model can then be described by the following differential equations:
$$\begin{cases} \frac{dS}{dt} = \pi - \mu S - \beta SI \\ \frac{dI}{dt} = \beta SI - \mu I - bI \\ \frac{dR}{dt} = bI - \mu R \end{cases} \tag{1}$$

The region of model (1) is $\Omega = \{(S, I, R) \epsilon \mathbb{R}_+^3 : S + I + R = N\}$, where N is constant. Since total population is constant, then $N = \frac{\pi}{\mu}$.

A key parameter in epidemiology is the basic reproduction number. Basic reproduction number, denoted by R_0, defined as the average number of new infections (secondary cases) transmitted by a single infected individual that is placed into a fully susceptible population (Diekmann, O. and Heesterbeek, J. A. P, 2000). In other words, R_0 tells us about the initial rate of spread of the disease. This value is also referred to as the treshold parameter (Anderson, R.M. and R.M. May, 1991). It is the most common measure for quantifying the strength of epidemics. If $R_0 < 1$, population is free from disease, otherwise if $R_0 > 1$, there will be an endemic in the population (Brauer, F., 2008). If $R_0 = 1$, the disease will remain in the population, but the number of infectious is not increase.

Analysis of the Political Figure Fever Transmission Model:

The local stable of each equilibrium point (critical point) is determined by the sign of eigenvalues for each critical point. Consider the case of two eigenvalues. If both the eigenvalues are real, then the equilibrium point is said to be a node and if they are conjugate complex numbers, then it is said to be a focus. If both the eigenvalues are positive, then the equilibrium point is an unstable node; if both the eigenvalues are negative, then it is a stable node and if one is positive and the other is negative, then it is a saddle point. For complex eigenvalues if the real part is positive, then the equilibrium point is an unstable focus and if the real part is negative, then it is a stable focus (Tapadar, A., 2011). Alternatively, the equilibrium point also can be shown by using determinant and trace from a Jacobian matrix for each rate change for each individual. The equilibrium point is locally stable if determinant is positive and trace is negative (Boyce, W.E. and R.C. DiPrima, 2001).

Consider that A is Jacobian matrix for $\frac{dS}{dt}$ and $\frac{dI}{dt}$
$$A = \begin{pmatrix} -\beta I - \mu & -\beta S \\ \beta I & \beta S - b - \mu \end{pmatrix}$$

Let (S^*, I^*, R^*) be the equilibrium of the model system (1) and define the basic reproduction number is $R_0 = \frac{\beta\pi}{\mu(\mu+b)}$. The model system (1) has two equilibrium points, which are as follows:

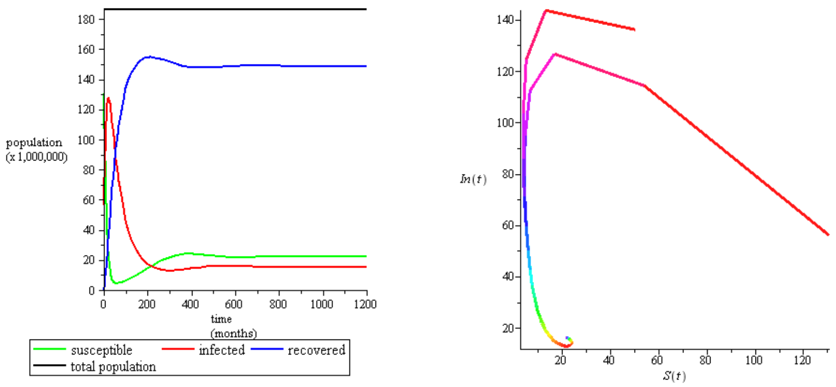
1. The political figure fever free equilibrium point
$$E_1\left(\frac{\pi}{\mu}, 0{,}0\right)$$
. Jacobian matrix for political figure fever free equilibrium point is
$$B = \begin{pmatrix} -\mu & -\beta\frac{\pi}{\mu} \\ 0 & \beta\frac{\pi}{\mu} - b - \mu \end{pmatrix}$$
with $\det(B) = -\pi\beta + \mu b + \mu^2$ and $\text{trace(B)} = -2\mu + \frac{\pi\beta}{\mu} - b$. If $\frac{\beta\pi}{\mu(\mu+b)} < 1$, then $\det(B) > 0$ and $\text{trace(B)} < 0$, therefore the political figure fever free equilibrium point is locally stable for $R_0 < 1$.
2. The political figure fever endemic equilibrium point
$$E_2\left(\frac{b+\mu}{\beta}, -\frac{-\pi\beta + \mu b + \mu^2}{\beta(b+\mu)}, -\frac{(-\pi\beta + \mu b + \mu^2)b}{\beta(b+\mu)\mu}\right)$$
Jacobian matrix for the political figure fever endemic equilibrium point is
$$C = \begin{pmatrix} \frac{-\pi\beta + \mu b + \mu^2}{b+\mu} - \mu & -b - \mu \\ -\frac{-\pi\beta + \mu b + \mu^2}{b+\mu} & 0 \end{pmatrix}$$
with $\det(C) = \pi\beta - \mu b - \mu^2$ and $\text{trace(C)} = \frac{-\pi\beta + \mu b + \mu^2}{b+\mu} - \mu$. The value of trace(C) is always negative. If $\frac{\beta\pi}{\mu(\mu+b)} > 1$, then $\det(C) > 0$, therefore the political figure fever endemic equilibrium point is locally stable for $R_0 > 1$.

Numerical Simulation of the Political Figure Fever Transmission Model:

In this section, the dynamical changes of each population with the transmission of political figure fever will be presented. The model presented above will be applied in case of Indonesian general election in 2014. The political figure that will be chosen here is Joko Widodo (Jokowi), one of the presidential candidates on Indonesian general election. For numerical simulation, we use parameter values as in Table 1. Total Indonesian people are 253,609,643 and total voters until November 4th, 2013 is 186,610,000 people (http://www.finance.detik.com (accessed 30 November 2013, 20:00)). Based on a survey before parliamentary elections, Jokowi gains 30% of the votes (http://www.republika.co.id (accessed 30 November 2013, 20:30)). The life span in the model is 48 years $\left(\frac{1}{\mu}\right)$ since we are looking at an voters age range of 17-65 years. Consider two scenarios for the rate of boredom, slow $\left(b = \frac{1}{60}\right)$ and fast $(b = 2)$.

Table 1: Parameter values.

Description	Symbol	Initial value	Units	References
Susceptible	S	130,627,000	People	(finance.detik.com)
Infected	I	55,983,000	People	(republika.co.id)
Recovered	R	0	People	Assumed
Recruitment rate	π	323,975.6944	People per month	Assumed
Transmission rate	β	0.00083	Per people per month	Assumed
Maturation rate	μ	1/576	Per month	Assumed
Boredom rate	b	{1/60,2}	Per month	Assumed



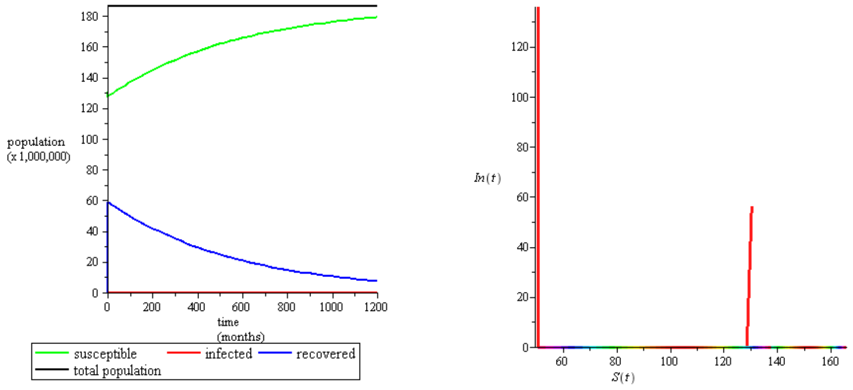
(a) (b)

Fig. 3: (a) Dynamical changes of each population (b) Phase portrait S-I $\left(b = \frac{1}{60}, R_0 = 8.416463094\right)$.

In figure 3 (a) and 3 (b), dynamical changes happened in every population in case of slow boredom $(R_0 > 1)$ can be seen. The solutions converge to the political figure fever endemic equilibrium state. From these figures, the number of infected increases while the number of susceptible decreases. Starting in the initial condition, decreasing population S was affected by movement population S to population I due to transmission or individual pull out himself from the system because of people don't use his/her rights to vote. After population S decrease, this population will increase and in the long term, population S will converge to 22 million individuals only. This increment happened due to a number constant new individual join into the population in bigger amount compare the movement population S to I or individual S who is pulled out from the system.

The population I moves up, almost linear, due to the increment of population S on transmission, although there is decreasing in individual move out from the system, or individual I who is moving to population R because individual I already bored with the political figure (i.e. in delivery vision and mission of campaign is not clear). After this, individuals I who move out from the system or individuals I who is moving to population R, not as big as individuals who is moving from population S to population I, so population I increasing. Population I will move down due to many individual I moves to population R and at the end in long term population I will converge to about 10 million individuals only.

This increasing movement also happened to the population R due to increasing population from I because individual I already bored with political figure, although there is individual R who move out from the system, but this amount not too big as individual who move from population I to population R. After that, the population R will decreasing little bit, because individual R who move from the system more than individual I who move to population R (due to slow boredom rate). At the end, in long term population R will converge to 150 million individuals. In long term, new political figures with new idea and new program will come out, so a lot of individual move to this new political figure, so population R become very big, compare other two populations (S and I). Therefore, it is very important for a political figure to keep their promises and execute the programs well.



(a) (b)

Fig. 4: (a) Dynamical changes of each population (b) Phase portrait S-I ($b = 2$, $R_0 = 0.07737598335$).

In Figures 4 (a) and 4 (b), dynamical changes happened in every population in case of fast boredom rate ($R_0 < 1$) can be seen. The solutions converge to the political figure fever free equilibrium state. The population S moves up and in the long term, this population will converge to the total population. This happened due to increment population S has been affected significantly with constant recruitment rate, meanwhile decreasing population S to I move slow, because fast boredom rate, so individual in population S who has been infected become population I will move fast to population R, therefore not much individual in population S can be infected by individual in population I because many people in population I has moved to population R.

From initial condition, population I shown move down dramatically and this population will be disappear very fast, due to high boredom rate which not followed with a fast transmission rate, so population I move fast to population R.

The population R moves up and then seems constant in 60 million populations. This increment happened due to fast movement from population I to population R because high boredom rate. Meantime, population R decreasing and in the long term this population converge to zero, because population S which has been infected become less, so population I who move to population R due to high boredom rate become less too.

In figures 3 (b) and 4 (b), dynamical changes in population S-I (phase portrait 2D) can be seen for different initial conditions. In the long term, for two different initial conditions, every population will converge to the same value. The number $R_0 > 1$ (Figure 3 (b)) describes that in the long term, population I will never disappear, but it will remain have some loyal supporters to their political figure. This is due to the continuous movement of population S to population I because of transmission, followed with slow movement from population I to population R because of small boredom rate. The number $R_0 < 1$ (Figure 4 (b)) describes that in the long term, population I will disappear (disease free), the population R will converge to zero, and population S will converge to the total population. This is happening due to fast boredom rate make the population I easily bored to a political figure or increasing maturation rate from each population significantly, therefore population S will filled by people who are not infected by a political figure.

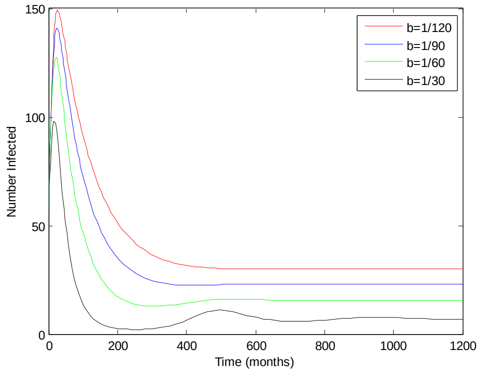


Fig. 5: Number of political figure fever with varying rate of boredom.

It can be seen from figure 5 that when the boredom rate of the political figure fever is higher, the infected individual by political figure will decrease faster. Consequently, the outbreak of the political figure fever will be eliminated and this will be a problem for supporting parties because they lose voters to choose their political figure, supporting parties must make strategies for keeping voters.

Conclusion:

In this paper, we derived and analyzed a simple deterministic model for the spread of voters in political figure fever. The model of political figure fever is studied through SIR disease transmission model approach. The basic reproduction number R_0 is found to classify the stability property of each equilibrium state. From numerical results, we conclude that if $R_0 < 1$ population is free from political figure fever, otherwise if $R_0 > 1$ there will be an endemic in the population. The dynamical behavior of each individual population for both conditions of the basic reproduction number has been presented in figure 3 and 4. It also can be seen that when there is the higher boredom rate between voter populations, the infected individual by political figure will decrease faster.

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